Technical Notes

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Effect of Thermal Stratification on Free Convection within a Porous Medium

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tion procedure. Numerical integration results are presented for a series of wall and ambient temperature distributions, which permit similarity solutions. As an extension of the preceding analysis, the second part of the study focuses on more practical problems, namely the conjugate conduction convection problems of a free convection fin embedded in a thermally stratified porous medium. It will be shown that the influence of the thermal stratification on the heat transfer is, in fact, quite significant.

Nomenclature

g = acceleration due to gravity
h = local heat transfer coefficient
K = permeability
m = parameter associated with thermal stratification
n,λ = power-law exponent of the wall temperature
Nux = local Nusselt number
q = local heat flux
Rax = local Rayleigh number
s = fin shape exponent
t = fin half-thickness
T = temperature
u = velocity component in the x direction
v = velocity component in the y direction
x,y = boundary-layer coordinates
α = thermal diffusivity
β = coefficient of thermal expansion
δ = scale for boundary-layer thickness
ν = kinematic viscosity

Subscripts

b = base
e = boundary-layer edge
f = fin
w = wall

Introduction

RECENTLY, convective heat transfer in a porous medium has begun to attract a great deal of attention in view of its applications to modern technologies such as underground heat exchangers for energy storage and recovery and temperature-controlled reactors.[1-3] Despite that possible thermal stratification may exist within a porous medium of finite extent, only a limited number of analyses[4,5] are available for certain special cases of thermal stratification.

In the first part of the present study, free convection over a vertical flat plate embedded in a thermally stratified porous medium is analyzed by exploiting the similarity transformation procedure.

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Similarity Solution for a Vertical Flat Plate

Referring to Fig. 1, which shows the physical model and its coordinate system, the boundary-layer equations governing the velocity and temperature fields, namely the equations of continuity, the Darcy's law, and the energy conservation, may be expressed as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial y}=\frac{K\beta}{\nu}g\frac{\partial T}{\partial y} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where the Boussinesque approximation is adopted along with usual boundary-layer approximations. Boundary conditions for the problem are given by

$$y=0: \qquad v=0 \tag{4a}$$

$$T=T_w(x) \tag{4b}$$

$$y\to\infty: \qquad u=0 \tag{4c}$$

$$T=T_e(x) \tag{4d}$$

The wall and ambient temperature distributions are assumed to follow a power function as

$$\Delta T_w \equiv T_w - T_e \propto x^\lambda \tag{5a}$$

$$T_e - T_c = m\Delta T_w \tag{5b}$$

where T_c is any constant reference temperature. The thermal stratification parameter m together with λ describe the ambient temperature variation such that $m=0$ and $m=-1$ correspond to a constant ambient temperature and a constant wall temperature, respectively. Since ΔT_w is positive everywhere and $dT_e/dx \geq 0$ for the surrounding fluid to be thermally stable, only the combinations of m and λ satisfying $m\lambda \geq 0$ are of physical interest.

The continuity equation [Eq. (1)] is automatically satisfied as we define the stream function:

$$u = \frac{\partial \psi}{\partial y} \tag{6a}$$

$$v = -\frac{\partial \psi}{\partial x} \tag{6b}$$

We now introduce the following transformations:

$$\psi = \alpha Rax^{1/2} f(\eta) \tag{7a}$$

$$T - T_e = \Delta T_w \theta(\eta) \tag{7b}$$

$$\eta = (y/x) Rax^{1/2} \tag{7c}$$

where

$$Rax = K\beta\Delta T_w gx/\nu\alpha \tag{8}$$

Rax is the local Rayleigh number while η is the similarity variable. The substitution of Eqs. (7) into Eqs. (2-6) yields

$$f'' = \theta' \tag{9a}$$

$$\theta'' + [(1+\lambda)/2] f\theta' - \lambda f'(\theta + m) = 0 \tag{9b}$$

with boundary conditions given by

$$\eta = 0: \quad f = 0 \tag{10a}$$

$$\theta = 1 \tag{10b}$$

$$\eta \to \infty: \quad f' = 0 \tag{10c}$$

$$\theta = 0 \tag{10d}$$

and the Darcian velocities are

$$u = (\alpha/x) Rax f' \tag{11a}$$

$$v = (\alpha/x) Rax^{1/2} \left(\frac{1-\lambda}{2} \eta f' - \frac{1+\lambda}{2} f \right) \tag{11b}$$

The primes in the foregoing equations denote the differentiations with respect to η. Equation (9a) may readily be integrated with Eqs. (10b) and (10d) as

$$f' = \theta \tag{12}$$

Substituting the foregoing equation into Eqs. (9b) and (10) yields

$$f''' + \frac{1+\lambda}{2} ff'' - \lambda f'(f' + m) = 0 \tag{13}$$

with boundary conditions, namely

$$\eta = 0: \quad f = 0 \tag{14a}$$

$$f' = 1 \tag{14b}$$

$$\eta \to \infty: \quad f' = 0 \tag{14c}$$

Equation (13) with the boundary conditions of Eqs. (14) may be solved by using any standard shooting procedure such as the Newton-Raphson method.

Once the temperature distribution is known, the local Nusselt number of our primary concern may be evaluated from

$$Nux \equiv q_w x/\Delta T_w k = [-\theta'(0)] Rax^{1/2} \tag{15}$$

where q_w and k are the local wall heat flux and the equivalent thermal conductivity of a porous medium, respectively. The heat transfer grouping $Nux/Rax^{1/2}$ is presented in Fig. 2, where the effects of the wall temperature distribution and thermal stratification on the local heat transfer rate may be seen. (When the values of x and ΔT_w are fixed, $Nux/Rax^{1/2}$ for different λ and m may be taken as the corresponding local wall heat flux q_w.)

Conjugate Conduction-Convection Analysis for a Vertical Fin

Pop et al.[6] obtained an exact solution for the conjugate problem of a vertical fin in a porous medium without considering its thermal stratification. In this study, we consider possible thermal stratification within the porous medium and derive a compact and useful expression for isotherms, upon exploiting an integral procedure successfully employed for Newtonian and nonNewtonian fluid flows (e.g., Refs. 7 and 8).

A schematical diagram of the infinitely long vertical fin is presented in Fig. 3. The upper closed end of the fin is attached to a base that is heated by an isothermal plate heat source embedded in a saturated porous medium of infinite extent. As

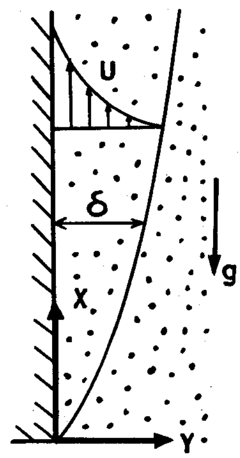

Fig. 1 Vertical flat surface in a porous medium.

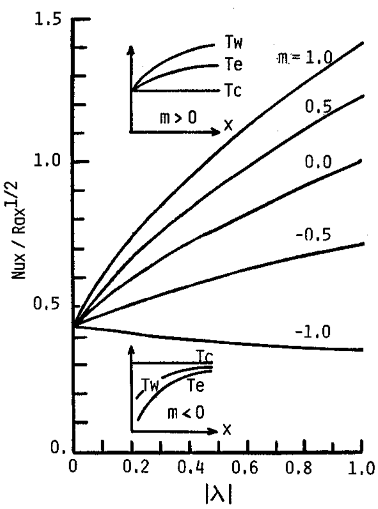

Fig. 2 Local heat transfer results.

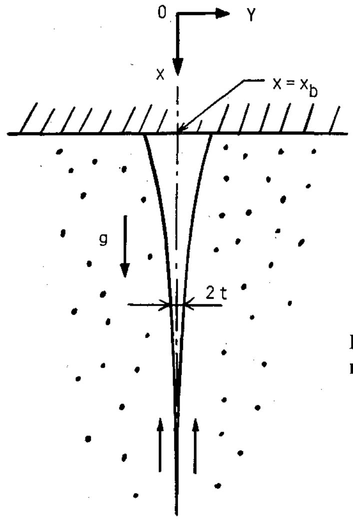

Fig. 3 Vertical fin in a porous medium.

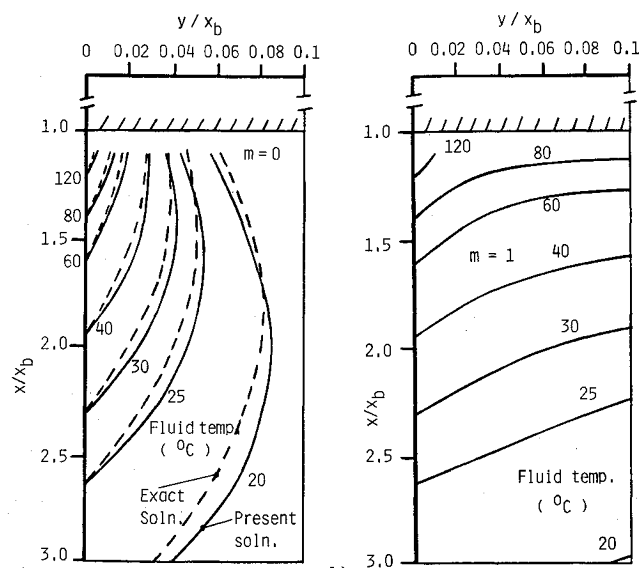

Fig. 4 Isotherms near a copper fin embedded in a geothermal reservoir; a) constant ambient temperature, b) variable ambient temperature ($m=1$).

in usual analyses of fins, the thin-fin approximation is adopted for the conduction through the fin. Thus, the fin temperature at any station x also serves as the wall temperature $T_w(x)$ for the adjacent porous medium. Due to the thermal stratification, the temperture of the surrounding porous medium $T_e(x)$ also varies in the vertical direction. Since the base temperature T_{w_b} $[=T_w(x_b)]$ exceeds that of the ambient porous medium [namely $T_{w_b}>T_w(x)>T_e(x)$], there is a convective fluid movement toward the fin base as a result of the buoyancy force. The coordinate x is taken downward along the center line of the fin while y is the coordinate normal to the fin. The origin of the coordinates is set such that the distance from the origin to the upper end of the fin is x_b. The value of x_b is not known a priori but is an outcome of the solution.

In addition to Eqs. (1-3), we now have to consider the fin energy equation, namely

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(tk_f\frac{\mathrm{d}T_w}{\mathrm{d}x}\right)+k\frac{\partial T}{\partial y}\bigg|_{y=0}=0 \tag{16}$$

where k_f is the thermal conductivity of the fin and t its half-thickness. The boundary conditions for the fin energy conservation equation (16) are

$$x=x_b: \qquad T_w=T_{w_b} \tag{17a}$$

$$x\to\infty: \qquad T_w\to T_e(\infty) \tag{17b}$$

Let us integrate Eq. (2) with the boundary conditions given by Eqs. (4) as

$$u=-(Kg\beta/\nu)(T-T_e) \tag{18a}$$

or

$$f\equiv u/u_w=(T-T_e)/\Delta T_w \tag{18b}$$

where

$$u_w\equiv-(Kg\beta/\nu)\Delta T_w \tag{18c}$$

Thus, the velocity profile is identical to the temperature profile. The following exponential profile may be assumed to prevail:

$$f(y/\delta)=\exp(-y/\delta) \tag{19}$$

where δ is not the conventional boundary-layer thickness but a scale for its thickness. Equation (19) automatically satisfies Eqs. (17).

The energy equation [Eq. (3)] together with Eq. (1) may be integrated under the conditions given by Eqs. (4):

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_0^\infty u(T-T_e)\mathrm{d}y+\frac{\mathrm{d}T_e}{\mathrm{d}x}\int_0^\infty u\mathrm{d}y=-\alpha\frac{\partial T}{\partial y}\bigg|_{y=0} \tag{20}$$

Upon substituting the assumed profile [namely Eq. (19)] into the foregoing integral equation, one obtains

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}x}\delta\Delta T_w^2+\delta\Delta T_w\frac{\mathrm{d}T_e}{\mathrm{d}x}=-\frac{\alpha\nu}{Kg\beta}\frac{\Delta T_w}{\delta} \tag{21}$$

It is interesting to note that similarity solutions exist when ΔT_w and T_e follow the functional forms as given by

$$\Delta T_w/\Delta T_{w_b}=(x/x_b)^{-n} \tag{22a}$$

$$T_e-T_c=m\Delta T_w \tag{22b}$$

where subscript b refers to the condition at the base.

The substitution of Eqs. (22) into Eq. (21) reduces Eq. (21) to an ordinary differential equation for δ^2 that may readily be solved as

$$(\delta/x)^2 Rax=4/[(3+4m)n-1] \tag{23}$$

provided that

$$n>1/3 \tag{24a}$$

where

$$Rax=Kg\beta\Delta T_{wx}/\alpha\nu \tag{24b}$$

Thus, the local Nusselt number $Nux=-(x/\Delta T_w)(\partial T/\partial y)_{y=0}$ of primary interest is given by

$$Nux/Rax^{1/2}=1/2\,[(3+4m)n-1]^{1/2} \tag{25}$$

The Nusselt number Nux may readily be calculated once the unknown exponent n is determined.

Equation (16) is now considered for the heat conduction through a thin fin under the assumption that the conductivity-thickness product varies according to

$$k_f t = (k_f t)_b (x/x_b)^s \tag{26}$$

where s is the prescribed fin shape exponent. The substitution of Eqs. (26), (23), and (19) into Eq. (16) yields

$$n(s-n-1)(k_f t/x^2) + \tfrac{1}{2}[(3+4m)n-1]^{1/2}(k\,Ra_x^{1/2}/x) = 0 \tag{27}$$

The foregoing equation suggests the existence of similarity solutions, namely

$$\left(\frac{kx}{k_f t}\right) Ra_x^{1/2} = \frac{kx_b}{(k_f t)_b} Ra_{x_b}^{1/2} = \frac{2(4-3s)(3-2s)}{[(3+4m)(3-2s)-1]^{1/2}} = Ncc \tag{28a}$$

where

$$n = 3-2s \tag{28b}$$

Therefore, the constraint described by Eq. (24a) is equivalent to

$$s < 4/3 \tag{29}$$

Once the thermal stratification parameter m and the fin shape exponent s for the conductivity-thickness product are given, the so-called "conduction/convection parameter" Ncc may readily be calculated from the last expression on the right-hand side of Eq. (28a). Thus, the unknown value x_b can be determined from Eq. (16a) as

$$x_b = \left[\left(\frac{\alpha\nu}{Kg\beta\Delta T_{w_b}}\right)^{1/2} \frac{(k_f t)_b}{k} Ncc\right]^{2/3} \tag{30}$$

The substitution of Eqs. (22), (23), and (28) into Eq. (19) leads to the following closed-form expression for the isotherms:

$$y/x_b = -\frac{kx_b/(k_f t)_b}{(4-3s)(3-2s)} \left(\frac{x}{x_b}\right)^{2-s} \ln\left[\left(\frac{T-T_c}{\Delta T_{w_b}}\right)\left(\frac{x}{x_b}\right)^{3-2s} - m\right] \tag{31}$$

In order to illustrate the temperature fields within the porous medium, the isotherms were obtained for the case of an infinitely long copper fin with constant thermal conductivity and thickness. The case was previously treated by Pop et al.[6] for the constant ambient temperature. Computations were carried out assuming $k_f = 376.8$ W/m°C, $t = 0.005$ m, $T_{wb} = 200$°C, $K = 10^{-8}$ m^2, $k = 2.428$ W/m°C, $g = 9.8$ m/s^2, $\beta = 1.8 \times 10^{-4}$/°C, $\nu = 0.27 \times 10^{-6}$ m^2/s, $\alpha = 0.63 \times 10^{-6}$ m^2/s, and the constant ambient temperature $T_e = 15$°C ($m = 0$). The resulting isotherms are plotted in Fig. 4a, which indicates a close agreement between the present solution and the exact solution.[6] As pointed out by Pop et al.,[6] the isotherms near the base curve back as a result of a strong flow acceleration there.

When the base is heated up to a high temperature, the effect of the thermal stratification within the porous medium may no longer be negligible. Thus, illustrative calculations were made with the same base temperature but with the thermal stratification parameter $m = 1$ and the reference temperature $T_c = T_e|_{x\to\infty} = 15$°C. Even for the same fin temperature distributions, the isotherms, as plotted in Fig. 4b, indicate a pattern quite different from that in Fig. 4a. These isotherms do not curve back toward the base but extend horizontally away from the fin surface as a result of the thermal stratification. In this sense, the previous solution for the constant temperature should be regarded as somewhat overidealized.

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Gas Particle Radiator

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Introduction

HIGH specific power (power radiated/radiator mass), small area, and long lifetime are the desirable characteristics of a space radiator. These characteristics will be attained if a low mass and high emissivity ϵ_T that is stable for long periods (7–10 years), can be achieved.

For a tube-type radiator (either a heat pipe or a pumped loop) high emissivity ($\epsilon > 0.8$) is achieved by the use of emissive coatings. Adhesion and emissive stability of these coatings must be obtained for long periods of time if a tube-type radiator is to be a successful space radiator. Generally, the largest mass portion of a tube radiator is the armor that must be used to protect it from meteoroid penetration.

The gas particle radiator (GPR) is a new concept that has the potential for a long lifetime and high emissivity with lower mass than tube radiators. Figure 1 is a conceptual drawing of the GPR. A gas which contains a suspension of fine particles is contained in a sealed volume between the tube radiator and an outer window that are separated by a distance, D. On start-up of the radiator, a temperature gradient will exist across the gas. This temperature gradient will induce a gas flow that will distribute the particles throughout the gas. However, this will have to be demonstrated for a successful GPR. In the

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